# The impact of High Resolution Spectral Analysis methods on the performance and design of millimetre wave FMCW radars

D. Bonacci[1], C. Mailhes[1], M. Chabert[1], F. Castanié[1]

1: ENSEEIHT/TéSA, National Polytechnic Institute, 2 Rue Camichel, BP 7122, 31071 Toulouse Cedex 7, France,
phone: +33 5 61 58 80 10, fax: +33 5 61 58 80 14,
email: david.bonacci, corinne.mailhes, marie.chabert, francis.castanie@tesa.prd.fr

**Abstract** This paper addresses the problem of joint measures of range and velocity of moving targets using millimetre wave FMCW radar (in the 77 Ghz range) within the field of automotive applications. The proposed solution is to determine range and velocity using spectral estimation of downconverted signals, theoretically composed of multiple sine functions embedded in noise. As a consequence, their accuracy is closely related to the accuracy of frequency estimation. In this paper, High Resolution spectral analysis methods (such as Auto-Regressive or Prony modeling) are shown to strongly impact the technological design constraints of the radars. More precisely, for a given sampling frequency of the downconverted signal, these methods show their ability either to significantly reduce the bandwidth of the linear frequency modulated radar sweeps although keeping constant the frequency resolution, or, for a given technological design, increase the same figure of merit. Moreover, adequate pre-processing of the signal is described, yielding correction of some 'nasty' non-linear effects (VCO, mixers, …) as well as denoising received signals. Theoretical study of the performances is given and illustrated on simulated and real signals (provided by the RadarNet project of the 5th Framework Program).

**Keywords** : High Resolution Spectral Analysis, millimetre wave radars, automotive applications.

## 1. Introduction

Within the field of an European project (http://www.radarnet.org/), automotive vehicules are equipped with FMCW (Frequency Modulated Continuous Wave) in order to prevent from collisions. The FMCW signals obtained after demodulation are linear combinations of sinusoids, the number of which is linked to the number of reflecting targets. Frequencies of these sinusoids carry both information of range and Doppler shift. Thus, accuracy and resolution on speed and range measurements are directly linked to those of frequency estimation.

Most widely used frequency estimation techniques in radar-data processing techniques are based on the (Fast) Fourier Transform and more particularly on the periodogram. This paper proposes the use of more sophisticated spectral estimation procedures (high resolution methods) as a way to improve frequency estimation and consequently range and velocity estimation. These procedures come from recent signal processing investigations and include parametric modeling [1] (as Autoregressive predictive methods) and subband decomposition [2-3]. In section 2, FFT-based and High-Resolution (HR) methods are discussed. Section 3 is devoted to subband decomposition and its interesting properties, especially for parametric spectral estimation. In section 4, simulation results are presented and commented. The last section gives conclusions.

## 2. Frequency estimation methods

Downconverted FMCW signals can be written as a sum of sinusoids with given amplitudes, phases and frequencies embedded in some additive noise. For range and velocity estimation, the main concern is the estimation of frequencies.

### 2.1 FFT-based methods

The main interest of classical periodogram-based methods is their low computational cost, since they use FFT (Fast Fourier Transform) algorithms. But these methods suffer from two important problems referred in [4] as short-range and long-range spectral leakage.

The first problem is the bias induced by the computation of FFT in a grid of discrete frequencies. Zero-padding is a way to mitigate this problem but it can be computationally expensive if high accuracy is needed. More adapted methods dealing with the granularity of the FFT have been developed, like the Interpolated FFT (IFTT) in [5-6], or the Weighted Phase Averager (WPA) method [7-8]. In these methods, a first coarse estimate of the frequency is corrected using phase information of the periodogram. WPA methods achieve, in general, better results than IFFT methods.

The second problem is long-range leakage. It denotes interference between sinusoidal components. It occurs even when using weighting windows. In [4], Santamar?a proposes an iterative version of WPA methods (IWPA) which is able to tackle both the short-range and long-range leakage problems. The basic idea is to subtract from the original signal all previously estimated components (once their frequencies, phases and amplitudes have been determined) before estimating a new component (with a new frequency). As a consequence, interferences are widely reduced and for each iteration, the use of a WPA method eliminates short-range leakage. The main difficulty of this method is that it requires very accurate initial frequency estimates because a small error in the frequency estimates could cause large errors in both the amplitude and phase estimates.

2.2 High resolution methods

These kinds of signals, i.e. a sum of complex exponentials or sinusoids embedded in a white additive noise, are particularly well-suited to parametric modeling. Among all existing parametric modeling methods [9-11], Auto-Regressive (AR) modeling is the most commonly used in parametric spectral analysis. This model assumes the signal under study *u(n)* to be a linear combination of its past samples plus an unexpected part *e(n)* :

$$u(n) = \sum_{k=1}^{p} a_k u(n-k) + e(n) \qquad (1)$$

The $a_k$'s are referred to as autoregressive parameters, *p* is the model order and *e(n)* is a white noise, corresponding to the Linear Prediction Error (LPE). AR modeling corresponds also to Linear Prediction as the above equation leads to consider that the signal *u(n)* is the output of a linear filter excited by a white noise. Thus, given *u(n)*, an AR model can be identified. Once the corresponding AR parameters are estimated, a spectral estimator of the signal *u(n)* can be proposed :

$$S_{AR}(f) = \frac{\sigma_e^2}{\left| 1 + \sum_{k=1}^{p} a_k e^{-i2\pi k f} \right|^2} \qquad (2)$$

The choice of the model order *p* is of great importance. Too low a guess for model order results in a highly smoothed spectral estimate. Too high an order introduces spurious details into the spectrum. Several criteria have been introduced as objective bases for selection of AR model order [9],[12].

Frequency estimation is done through the estimation of the AR polynomial roots – polynomial coefficients are the AR parameters. This estimation method leads to better results than applying a “peak-detection” algorithm on the AR spectrum (2). Indeed, the main drawback of peak-detection algorithms is the necessary choice of a threshold in order to distinguish signal components from noise ones. Moreover, the estimation of AR polynomial roots can be done in real-time. For example, in the Bairstow technique [13], the complex roots of a real polynomial are calculated by finding real quadratic factors. Other algorithms, as in [14] are based on a continued fraction representation of the rational transfer function. As a consequence, the application of a peak-detection algorithm, and the sensitive choice of a threshold is no more necessary when using HR methods.

## 3. Subband Decomposition

Subband decomposition is an operation consisting in filtering an analyzed discrete signal *u(n)* through an adapted filterbank and then decimating (i.e. keeping only one sample out of several samples) the obtained filtered signals. This can be schematized by the following figure, showing the case of an uniform filterbank (same decimation factor on each branch):

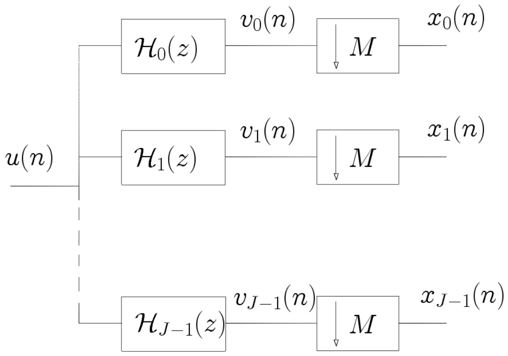


**Figure 1 : Uniform Analysis Filterbank**

The main idea is that, when using an ideal filterbank (infinitely sharp bandpass filters), it is possible to easily reconstruct the fullband spectrum $S_u(f)$, subband after subband, as a function of the spectra of the subband signals $x_j(n)$. Obviously, this algorithm can be written in a parallel way, allowing the spectrum estimation on *M* simultaneous frequency subbands, using the spectral estimation procedure on each branch of the filterbank. For further details about subband decomposition, the reader can to refer to [2].

From a spectral analysis point of view, subband decomposition has several advantages and drawbacks. The main benefits provided by subband decomposition can be expressed as follows in the case of parametric spectral estimation: model order reduction and consequently condition number decreasing for autocorrelation matrices [15], spectral density whiteness and also linear prediction error power reduction for autoregressive (AR) estimation [16]. In the case of a peaked spectrum signal (case of FMCW radar signals), another very interesting property can be pointed out: frequency spacing and local Signal to Noise Ratio (SNR) increase by the decimation ratio [17]. Another great feature of subband decomposition is the ability to save computational time for most parametric spectral estimation procedures: applying *M* times an algorithm in the subbands with order *p/M* is often more efficient than applying the same algorithm once on the fullband signal, with order *p*.

Unfortunately, there are also drawbacks when doing parametric spectral estimation on subbands. The first one is spectral overlapping: when using non-ideal filterbanks, the same harmonic component may appear in two contiguous subbands at two different frequencies. The second one is the relative variance increase for autocorrelation estimators, due to sample number decrease after decimation. The first drawback has already been addressed in two recent papers [18] and [19] where (non real-time) procedures were proposed to perform subband spectral estimation without discontinuities or aliasing, even at subbands borders. Within the RadarNet context, these procedures were not used as they are not real-time. As aliasing occurs at subband borders, frequencies lying in these areas are actually estimated by modulating the input signal prior to its subband decomposition. The second drawback is mitigated because of the use of an order *p/M* in the subbands while using an *M*-fold decimator.

## 4. Simulation results

In the RadarNet's signal processing chain, main concern is to estimate as precisely as possible range and speed of multiple moving targets. This is done using two radar FMCW consecutive sweeps (rising then falling edges) as in figure 2.

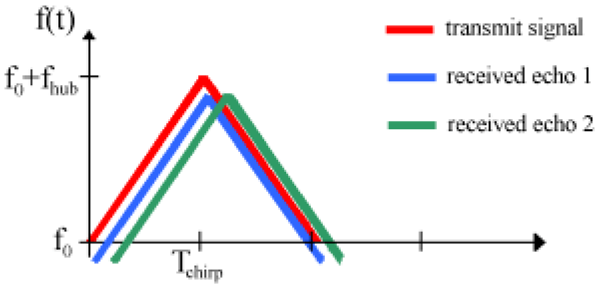


**Figure 2 : Instantaneous frequency for a sample pattern of 2 chirps**

Considering a single target, we denote by $f_1$ its corresponding detected frequency for chirp 1 and $f_2$ for chirp 2. Then range $R$ and velocity $v$ are given by the following relationships:

$$R = -\frac{CT_{chirp}}{4f_{hub}}(f_1 - f_2) \quad (3)$$

$$v = -\frac{C}{4f_0}(f_1 + f_2) \quad (4)$$

where each linear FMCW sweep begins at frequency $f_0$, with bandwidth $f_{hub}$. $T_{chirp}$ is the duration of a single measurement and $C$=3.10$^8$ *m/s*. Two different specifications are usually imposed on both range and speed: accuracy *dR*, *dv* and resolution *DR*, *Dv*. Eq. (5) and (6) show that these constraints are directly linked to frequency accuracy *df* and resolution *Df*.

$$\delta R = -\frac{CT_{chirp}}{2f_{hub}}\delta f, \;\; \Delta R = -\frac{CT_{chirp}}{2f_{hub}}\Delta f \quad (5)$$

$$\delta v = -\frac{C}{2f_0}\delta f, \;\; \Delta v = -\frac{C}{2f_0}\Delta f \quad (6)$$

In the two preceding equations, *d* holds for accuracy and *D* for resolution. In the rest of this section, High Resolution (HR) methods are compared to FFT-based methods for frequency estimation taking into account both accuracy and resolution. Accuracy will be measured via the Mean Square Error (MSE) (this figure of merit takes into account both bias and variance) and resolution will be given by the ability to separate two closed frequencies. Note that a good theoretical figure of merit for frequency resolution is the bandwidth of the peak at –3 dB. Except for simulation on a real signal (provided by the RadarNet research project), the analyzed signal is a sum of $K$ sinusoids corrupted by an additive gaussian white noise $b(n)$:

$$u(n) = \sum_{m=1}^{K} A_m \cos(2\pi f_m n + \phi_m) + b(n) \quad (7)$$

where $K$ denotes the number of sinusoids, the amplitudes $A_m$ and phases $f_m$ are not to be determined (except if using the IWPA method) and $f_m$ are the normalized frequencies under interest. Signal to Noise Ratio (SNR) is defined as follows:

$$SNR = \frac{\sum_{m=1}^{K} A_m^2/2}{\sigma_b^2} \quad (8)$$

For the following simulations, the IWPA method presented in section 2 is used as a FFT-based method. For HR method, a Fast Least Square Algorithm [20] is applied on signals at the output of an uniform filterbank with *J=M=20* subbands. The computational complexity of this algorithm is $10(N-p)p$ operations. Then using subband decomposition and an order *p/M* in each subband, this complexity is reduced to $M10\left(\frac{N}{M}-\frac{p}{M}\right)\frac{p}{M} = 10(N-p)\frac{p}{M}$. Even more computational power can be saved if not all subbands have to be analyzed. For instance, in the RadarNet project, all normalized frequencies under interest lie in the interval [*0, 0.1*] and only the first 5 subbands are used. Filters corresponding to these are depicted on figure 3.

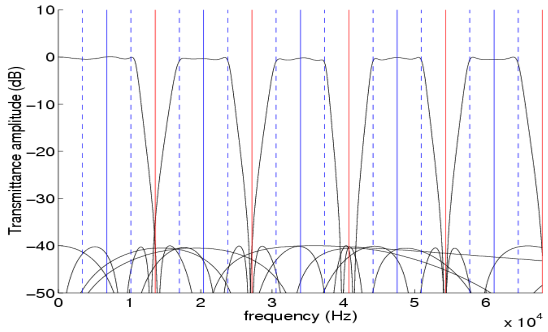


**Figure 3 : Uniform filterbank with *M*=20 subbands**

As can be seen, non-overlapping filters are used. By the way, there is no problem of spectral overlapping but frequencies lying at subbands borders have to be previously modulated by a frequency shift corresponding to half a subband width.

### 4.1 Frequency accuracy

Simulations are done with *500* independent trials on signals of the form (7). Used parameters were *N=1024* samples, *K=2* sinusoids with $A_1 = A_2 = 1$ and *SNR=3* (corresponding to *4.77* dB). Chosen modeling order for subband AR estimation was *p=7*. In order to measure accuracy, relatively well-separated frequencies were chosen: $f_1=0.04$ and $f_2=0.08$. The phases $f_1$ and $f_2$ are uniformly distributed on [*0, 2p* [.

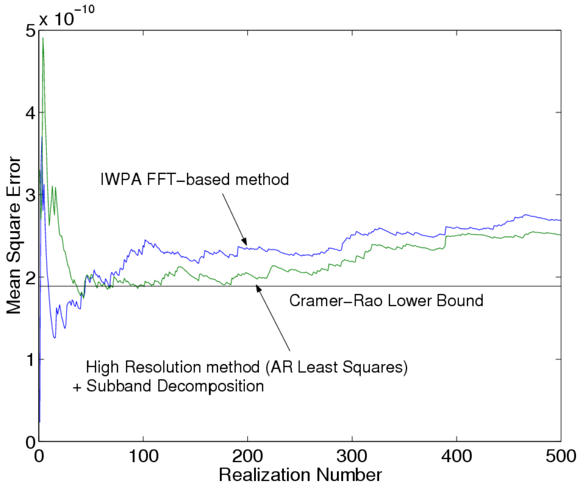


**Figure 4 : MSE on $f_1$ estimation using HR methods and FFT-based methods (well-separated frequencies)**

Both frequency estimator (HR and FFT-based) are unbiased. Theoretically speaking, it can be found in [11], p. 106, the demonstration that the frequency estimator $\hat{f}$ obtained by finding the roots of the AR polynomial is such that $\sqrt{N}(\hat{f} - f)$ has an asymptotic zero-mean Gaussian distribution. Moreover, figure 4 shows that their variances reach the Cramer-Rao lower bound. It follows that HR methods and FFT-based methods are quasi-equivalent for accuracy ***df*** of frequency estimation (at first and second orders).

4.2 Frequency resolution

AR spectral estimator has to be compared to those obtained based on FFT-techniques from a spectral resolution point of view. The maximum frequency resolution using FFT-based methods is of the form:

$$\Delta f_{FFT} \simeq \frac{1}{N} \tag{9}$$

*N* corresponding to the length of the observed signal given in (7) and $Df_{FFT}$ corresponding to the width of the spectral peak at –3dB level. Dealing with AR modeling, it can be shown that in the case of a sinusoidal signal embedded in a white noise, the spectral resolution can be written [20]:

$$\Delta f_{AR} \simeq \frac{6}{\pi p\,(p+1)\,SNR} \tag{10}$$

where SNR represents the Signal to Noise Ratio of the signal under study. The model order can be considered as being a fraction of the signal length, i.e., proportional to *N*. Based on this remark, it is obvious that AR modeling can bring better spectral resolution since $Df_{AR}$ is inversely proportional to $N^2$ while $Df_{FFT}$ is inversely proportional to *N*. However, it can be worse also, due to the SNR term.

Simulations are done with the same parameters as above, with 500 independent trials. The only difference is that frequencies are now chosen to be more closed than the maximum FFT resolution *1/N*: $f_1=0.04$ and $f_2=0.04+0.7/1024$. The obtained results are depicted on figure 5.

Theoretical considerations and simulations show the interest of HR methods for frequency resolution *Df*. At *SNR=4.77* dB (which is realistic with respect to real radar sensors signals), the use of HR methods allows a gain by a factor of more than 3. Hence, for a given range and velocity resolution *DR* and *Dv*, equations (5) and (6) show that the initial frequency $f_0$ and excursion frequency $f_{hub}$ could be reduced by the same factor, which can strongly impact the technological design constraints of the radars.

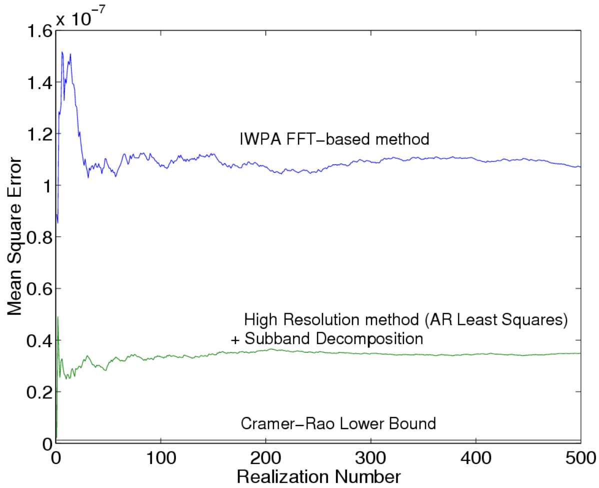


**Figure 5 : MSE on $f_1$ estimation using HR methods and FFT-based methods (closed frequencies)**

4.3 Real signal

Real downconverted signals suffer from non linear effects resulting in time-ramps on the collected time data. As a consequence, the signal model (7) is no more theoretically valid. Figure 6 shows the case of 2 real records provided by the RadarNet team (two above plots).

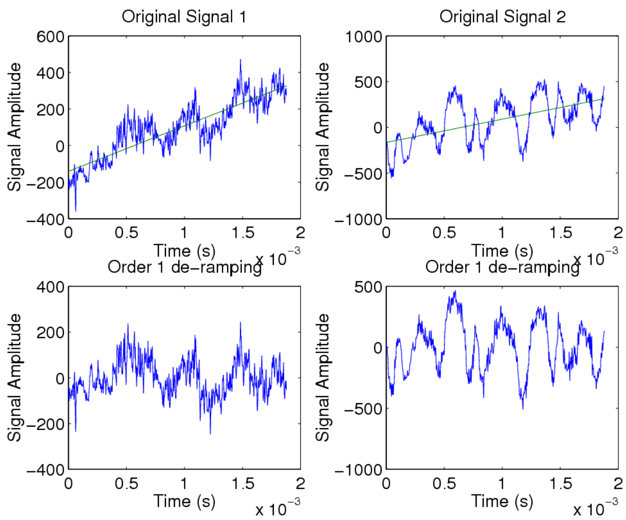


**Figure 6 : Order 1 de-ramping**

As HR methods are highly sensible to this kind of non-stationarity, a pre-processing step was proposed, consisting in fitting an order 1 polynomial in a least-square sense to the raw data. The result of this de-ramping procedure is shown on figure 6.

Using this pre-processing step, figure 7 shows the FFT and AR spectra versus normalized frequencies, reconstructed from subband spectral estimation, for a particular radar record of length $N=1024$ samples. An order $p=30$ was used with a $M=20$ uniform filterbank.

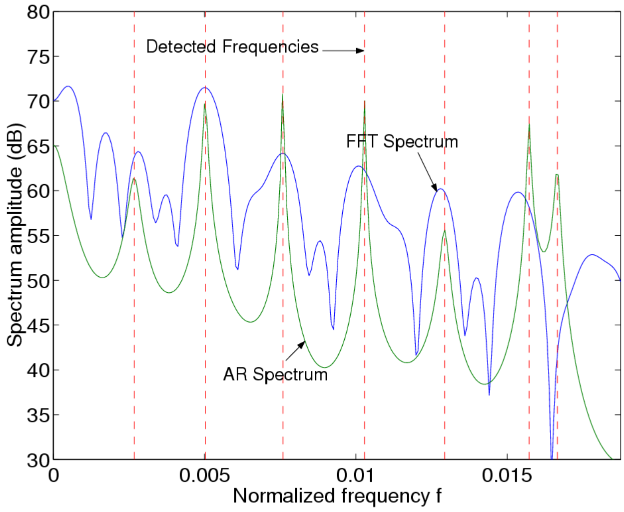


**Figure 7 : HR and FFT-based spectra for a real radar signal**

It can be seen on the last figure that HR methods used conjointly with an appropriate subband decomposition and pre-processing of the time signal leads to a very important resolution improvement with respect to the periodogram.

### 5. Conclusion

High Resolution Analysis Techniques and Fast Fourier Transform algorithm have been compared in the context of automotive radar signal processing, taking into account both range and speed accuracy and resolution. An auto-regressive model has been proposed, preceded by a de-ramping stage and subband decomposition. The frequency resolution improvement has been demonstrated through simulations and theoretical considerations for a parametric method based on a least square resolution. This results on a technological design ease as the initial frequency $f_0$ and excursion frequency $f_{hub}$ of the radar sweeps can be reduced while keeping same speed and range resolution. Moreover, it has been stated that computational cost could be reduced thanks to the use of subband decomposition.

### 6. Acknowledgment

The authors are indebted to the European 5$^{th}$ Framework Program for its financial support thru the RadarNet program, and the Prof. Rohling's team of the University of Hamburg for providing the radar actual signals.

### 7. References

[1] S. M. Kay: "*Modern Spectral Estimation: Theory and Applications*", Prentice Hall Signal Processing Series, 1988.

[2] P.P. Vaidyanathan: "*Multirate Systems and Filterbanks*", Prentice Hall Signal Processing Series, 1993.

[3] R.E. Crochiere and L.R. Rabiner: "*Multirate Digital Signal Processing*", Prentice Hall Signal Processing Series, 1983.

[4] I. Santamaria, C. Pantaleón and J. Ibañez: "*A comparative study of high-accuracy frequency estimation methods*", Mechanical Systems and Signal Processing, vol. 14, no. 5, pp. 819-834, Sept. 2000

[5] D.C. Rife and G.A. Vincent: "*Use of the discrete Fourier Transform in the measurement of levels and tones*", Bell System Technical Journal, 49, 197-228, 1970.

[6] V.K. Jain, W.L. Collins and D.C. Davis: "*High-accuracy analog measurements via Interpolated FFT*", IEEE Transactions On Instrumentation and Measurement, 28, 113-122, 1979.

[7] S.M. Kay: "*A fast and accurate single frequency estimator*", IEEE Transactions On Acoustics, Speech and Signal Processing, 37, 1987-1990, 1989.

[8] S.A. Tretter: "*Estimating the frequency of a noisy sinusoid by linear regression*", IEEE Transactions On Information Theory, 31, 832-835, 1985.

[9] A. Ducasse, C. Mailhes and F. Castanié: "*Estimation de fréquences: panorama des méthodes paramétriques*", Traitement du Signal, 15, 149-162, 1998.

[10] S.L. Marple: "*Digital spectral analysis with applications*", Prentice Hall Signal Processing Series, 1988.

[11] F. Castanié et al.: "*Analyse spectrale*", Hermès, Collection IC2, 2003.

[12] B. Porat: "*Digital processing of random signals*", Prentice Hall Signal Processing Series, 1994.

[13] M.J. Maro et al.: "*Numerical Analysis: A Practical Approach*", New York Macmillan, 1982.

[14] W.B. Jones and A.O. Steinhardt: "*Finding the poles of the lattice filter*", IEEE Transactions On Acoustics, Speech and Signal Processing, 33, 1328-1331, 1985.

[15] G. H. Golub and C. F. Van-Loan: "*Matrix Computations*", The Johns Hopkins University Press, 1989.

[16] S. Rao and W. A. Pearlman: "*Analysis of linear prediction, coding, and spectral estimation from subbands*", IEEE Transactions On Information Theory, 42, 1160-1178, 1996.

[17] A. Tkacenko and P. P. Vaidyanathan: "*The role of filter banks in sinusoidal frequency estimation* J. Franklin Institute journal, 338, 517-547, 2001.

[18] D. Bonacci, P. Michel and C. Mailhes: "*Subband decomposition and frequency warping for spectral estimation*", EUSIPCO, Toulouse, France, 147-150, 2002.

[19] D. Bonacci, C. Mailhes and P. M. Djuric: "*Improving frequency resolution for correlation-based spectral estimation methods using subband decomposition*", ICASSP, Hong-Kong, China, 6, 329-332, 2003.

[20] J.L. Lacoume: "*Close frequency resolution by maximum entropy spectral estimatorsr*", IEEE Transactions On Acoustics, Speech and Signal Processing, 32, 977-983, 1984.