# Improving High Resolution Spectral Analysis methods for Radar measurements using Subband Decomposition

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***Abstract*—This paper addresses the problem of spectral analysis on radar measurements using high resolution methods. These methods have already been shown to yield better results than Fast Fourier Transform (FFT) based methods for accuracy on detected frequencies and more particularly for frequency resolution. In most applications, these performances are closely related to the performances of range and velocity estimation. In the paper, theoretical study shows the interest of subband decomposition for improving performances of frequency estimation in the case of the use of High Resolution methods, while it is shown to be inefficient when using FFT-based algorithms. Some elements of computational cost are given, in order to compare fullband and subband processing when using Fast Least Square Autoregressive (AR) algorithm. Finally, experimental results are given, showing the interest of subband decomposition within the frame of radar signal processing either for accuracy and resolution on frequency estimation.**

***Index Terms*— Subband decomposition, radar, high resolution, parametric modelling, spectral analysis.**

## I. Introduction

WITHIN the field of the European project RadarNet (http://www.radarnet.org/), FMCW (Frequency Modulated Continuous Wave) radars are mounted on automotive vehicules in order to carry out different applications (such as collision warnings or parking aid). Signals obtained after demodulation are combinations of sinusoids and their frequencies carry both information of range speed. Accuracy and resolution on range and speed estimates are directly linked to those of frequency estimation and this paper proposes subband decomposition as a way to improve the performances of spectral analysis when using parametric modelings. Even when not using subband decomposition, the use of parametric modelings (High Resolution methods) has already been shown to yield better results than Fast Fourier Transform (FFT)-based techniques. Section II presents the interest of High Resolution methods in the context of frequency estimation for radar signals and section III is devoted to subband decomposition. Simulation results are presented in section IV and conclusions are reported in section V.

## II. Interest of parametric modelings for frequency estimation

Within the RadarNet context, the FMCW radar signals obtained after demodulation are linear combinations of sinusoids embedded in noise. Therefore, these signals can be written as :

$$u(t) = \sum_{m=1}^{K} A_m \cos(2\pi f_m t + \phi_m) + n(t), \qquad (1)$$

where $K$ denotes the number of reflecting targets, $n(t)$ represents some additive noise (pink noise = $1/f$ noise in the case of RadarNet sensors) and $A_m$ is the amplitude of a given component. The frequencies $f_m$ carry both information of range and Doppler shift allowing to derive targets velocity.
Auto-Regressive (AR) modelling is the most commonly used in parametric spectral analysis. This model assumes the discrete signal under study $u(n)$ to be a linear combination of its past samples plus an unexpected part $e(n)$:

$$u(n) = \sum_{k=1}^{p} a_k u(n-k) + e(n), \qquad (2)$$

where $p$ is the model order. The choice of $p$ is of great importance and several criteria have been introduced as objective bases for selection of AR model order [1], [2]. Once the corresponding AR parameters $a_k$ are estimated, a spectral estimator of the signal $u(n)$ can be proposed:

$$S_{AR}(f) = \frac{\sigma_e^2}{\left|1 + \sum_{k=1}^{p} a_k e^{-i2\pi kf}\right|^2}. \quad (3)$$

The estimation of the AR polynomial roots leads to frequency estimation: estimated frequencies can be easily derived from the roots of the denominator of (3). Compared to FFT-based methods, for which frequency estimation is done using peak-detection algorithm, the sensitive choice of a threshold in order to distinguish signal components from noise ones is no more necessary. Obviously, the estimation of AR polynomial roots can be done in real-time using, for instance, the Bairstow technique [3]. In this method, the complex roots of a real polynomial are calculated by finding real quadratic factors. Other algorithms, as in [4], are based on a continued fraction representation of the rational transfer function.

From a spectral resolution point of view, the maximum frequency resolution using FFT-based methods is of the form:

$$\Delta f_{FFT} \simeq \frac{1}{T_{chirp}}, \quad (4)$$

$T_{chirp}$ corresponding to the length of the observed signal given in (1) and $\Delta f_{FFT}$ corresponding to the width of the spectral peak at -3dB level. Dealing with AR modelling, it can be shown that in the case of a sinusoidal signal embedded in a white noise, the spectral resolution can be written [5]:

$$\Delta f_{AR} \simeq \frac{6}{\pi p \,(p+1)\, SNR}, \quad (5)$$

$p$ being the model order and SNR the Signal to Noise Ratio of the signal under study. Hence, for not too low SNRs, there can be an important gain when using High Resolution methods for well chosen model order $p$.

## III. Subband Decomposition

Although some authors use subband decomposition to improve classical spectral estimation (based on the Fourier transform) [6], it is more potent when it is applied in combination with parametric spectral estimation methods [7], [8]. In these papers, subband spectral estimation is shown to yield better performances than applying spectral estimation on the original fullband process. This has been shown for a bank of ideal infinitely sharp bandpass filters. However, some experimental results have highlighted that the improvements brought by subband spectral estimation still remains in the case of non-ideal filterbanks such as modified Quadrature-Mirror Filters (QMF's) or cosine modulated filterbanks [8]. Within the field of frequency estimation, benefits of subband decomposition for order selection are illustrated in [9] in the case of two separate narrow peaks.

Thus, the performance of traditional or parametric spectral estimation methods can be improved when applied to signals filtered by an appropriate filterbank rather than applied to the corresponding fullband signal. The list of benefits provided by subband decomposition is as follows:

- Model order reduction and consequently condition number decreasing for autocorrelation matrices [10].
- Frequency spacing and local Signal to Noise Ratio (SNR) increase by the decimation ratio (for signals composed by a sum of sinusoids corrupted by additive noise) [11].
- Whitening of noise in the subbands [11].
- Linear prediction error power reduction for AR estimation [7].

Obviously, subband spectral estimation has also some drawbacks:

- Spectral overlapping (aliasing): when using non-ideal filterbanks, the same harmonic component may appear in two contiguous subbands at two different frequencies.
- Relative variance increase for autocorrelation estimators (due to decimation).

The first drawback has already been addressed in two recent papers [12] and [13]. Induced frequency overlapping may be troublesome, bringing edge effects at subband borders and, in these papers, non-realtime methods were proposed that allow for subband spectral estimation without any problem of discontinuity or spectral overlapping at subband borders. In a realtime context, these procedure are not useable and another solution must be considered.

Aliasing occurs due to decimation of the signals by the filterbank. Considering the $j^{th}$ branch of the bank represented in figure 1 , the Fourier Transform of the decimated signal versus the input of the filter bank $u(n)$ can be written as:

$$X_j(e^{i2\pi Mf}) = \frac{1}{M}\sum_{k=0}^{M-1} \mathcal{H}_j(e^{i2\pi(f-\frac{k}{M})})U(e^{i2\pi(f-\frac{k}{M})}). \quad (6)$$

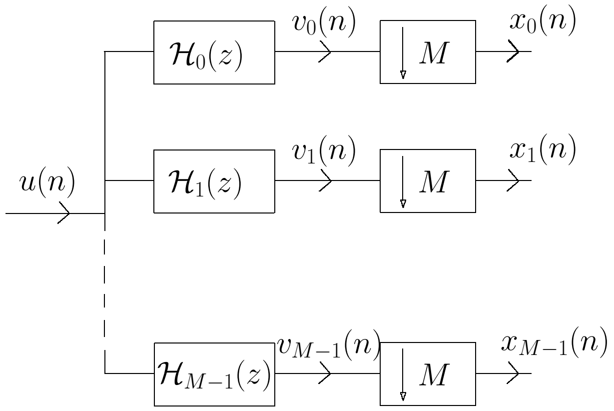


Fig. 1. Uniform filterbank with $M$ subbands.

This expression highlights spectral overlapping given by the terms for $k = 1, ..., M-1$. Obviously, ideal infinitely sharp bandpass filters bring no aliasing. In order to cancel aliasing in practical conditions, quasi non-overlapping filters were used as in figure 2. Actually, the filters overlap as it can be seen in a semi logarithmic scale but the use of such a filterbank makes it possible to detect peaks with amplitude greater than $-40dB$.

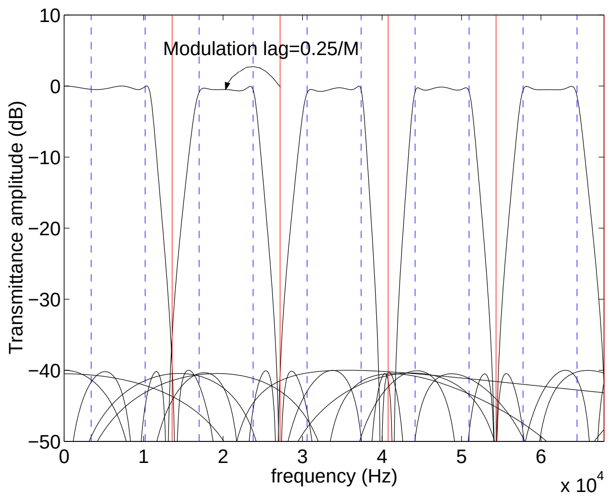


Fig. 2. Filterbank transmittance for $M = 20$ subbands.

Each subband has a frequency width equal to $0.5/M$ (normalized frequency). The use of such a filterbank allows to estimate frequencies lying in the middle of the subbands with no problem of aliasing (not overlapping filters). For frequencies lying near the subband border, a pre-processing step has been proposed in order to bring back frequencies of interest in the middle of their corresponding subband. It consists in modulating the input signal $u(n)$ by a quantity of $0.25/M$ (half a subband width) and lowpass filtering. The obtained signal $y(n)$ is then passed through the filterbank and frequency estimation can be performed in the middle of the subband (where there is no aliasing). More precisely, the modulated signal $u_\Delta(n)$ is obtained using the following relationship:

$$u_\Delta(n) = u(n)\cos\left(2\pi\frac{0.25}{M}n\right). \tag{7}$$

Hence the spectrum of signal $u_\Delta(n)$ expresses as:

$$S_{u_\Delta}(f) = \frac{1}{4}\left[S_u(f - \frac{0.25}{M}) + S_u(f + \frac{0.25}{M})\right]. \tag{8}$$

After lowpass filtering with well chosen cut-off frequency, it remains signal $y(n)$ with spectrum:

$$S_y(f) = \frac{1}{4}S_u(f + \frac{0.25}{M}), \tag{9}$$

for frequencies lying in the relevant subband. The true normalized frequency is then derived by adding $0.25/M$ to the frequency detected thanks to signal $y(n)$.

## IV. Simulation Results

For HR method, a Fast Least Square Algorithm [14] is applied on signals at the output of an uniform filterbank with $M = 20$ subbands (the frequency response is given figure 2). The computational complexity of this algorithm is $10(N-p)p$ operations. Then using subband decomposition and an order $p/M$ in each subband, this complexity is reduced to $M10(\frac{N}{M} - \frac{p}{M})\frac{p}{M} = 10(N-p)\frac{p}{M}$. Even more computational power can be saved if not all subbands have to be analyzed. Knowing the AR parameters, the estimation of the AR polynomial roots leading to frequency estimation is done using the Bairstow algorithm [3].

Using these processing techniques, figure 3 shows the FFT and AR spectra versus normalized frequency, reconstructed from subband spectral estimation, for a particular radar record of length $N = 1024$ samples. An order $p = 30$ was used with a $M = 20$ uniform filterbank. It can be seen on figure 3 that HR methods used conjointly with an appropriate subband decomposition leads to a very important resolution improvement with respect to the periodogram.

In order to show the improvement brought by subband decomposition in terms of accuracy, the analyzed signal is now assumed to be a pure sinusoid embedded in white noise with a random phase $\phi$ uniformly distributed between 0 and $2\pi$.

$$u(n) = A\sin(2\pi f_0 n + \phi) + b(n), \tag{10}$$

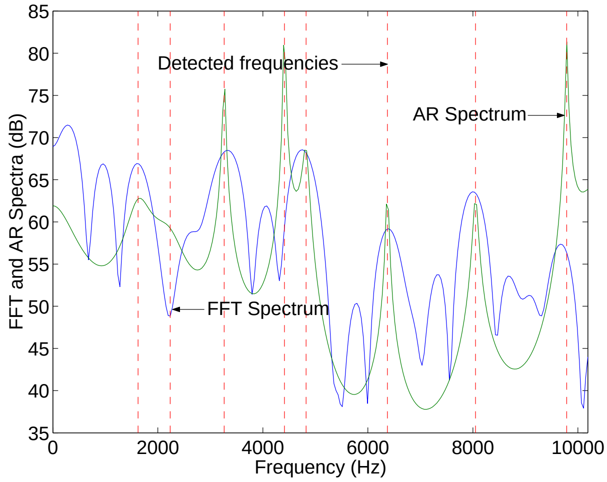


Fig. 3. Comparison FFT vs AR spectra for resolution.

where $A = 1$, $f_0 = 0.1$ (normalized frequency) and $b(n)$ is a white noise with power $\sigma_b^2 = 0.05$ ($SNR = 10\ dB$). The variance on $f_1$ estimation is plotted in figure 4 when subband or fullband AR estimation is performed. This figure shows that the

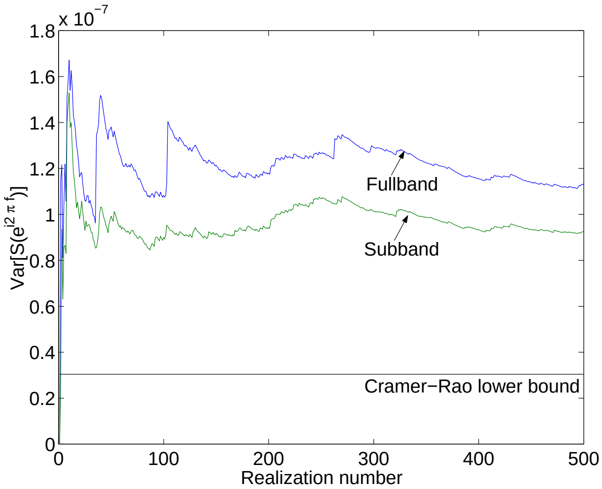


Fig. 4. Estimation variance at $f_1$ versus the realization number.

variance of the frequency estimates is smaller with subband decomposition.

## V. Conclusions

The aim of this paper was to propose subband decomposition as a way to improve spectral estimation and more particularly frequency estimation in the context of radar signal processing. When using High Resolution methods (as AR modelling), which has already been shown to yield better results than FFT-based algorithms, it has been pointed out that subband decomposition allows to reach better performances (in terms of accuracy and resolution) on detected frequencies. This improvement obviously results in a gain in accuracy and resolution for distance and velocity. Moreover, it has been stated that computational cost could also be reduced.

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